S-Plus functions for truncated and weighted means of asymmetric distributions:

Weibull, Gamma, and Lognormal cases

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Abstract

Marazzi and Ruffieux (1998, 1999) introduce the truncated mean, the approximate quartile based truncated mean, and the weighted mean estimates of the mean of an asymmetric distribution. Marazzi (2001) defines smoothed versions of these procedures in order to compute their influence functions when the underlying distribution is discrete. This manual describes S-plus functions for the computation of the estimates, their influence functions, and their asymptotic variances for the Weibull, the Gamma, and the Lognormal distributions.

S-Plus functions

The notations of Marazzi and Ruffieux (1998, 1999) and Marazzi (2001) are used. The references to these papers are abbreviated: MR98, MR99, and MA01. The papers can be obtained by writing to `Alfio.Marazzi@unil.ch`.

The following models for the distribution of a positive random variable X are considered:

- the Weibull distribution with parameters α and σ, where α is the location and σ the scale parameter of the distribution of $\ln(X)$; the shape parameter of the Weibull distribution is $\vartheta_1 = 1/\sigma$ and the scale parameter is $\vartheta_2 = \exp(\alpha)$;
- the Gamma distribution with shape α and scale σ;
- the Lognormal distribution with normal mean α and normal scale σ.

The following functions are described.

Weibull case

`D.E.weibull`	LD_D-estimates of shape, scale, and mean.
`TD.weibull`	Truncated mean.
`WD.weibull`	Weighted mean.
`ATR.weibull`	Approximate quartile based truncated mean.
`AV.DE.weibull`	Asymptotic variance of the LD_D-estimate.
`AV.TD.weibull`	Asymptotic variance of the truncated mean.
`ACV.DE.weibull`	Asymptotic covariance of the LD_D-estimate.
`ACV.TD.weibull`	Asymptotic covariance of the TD-estimates.
`AV.WD.weibull`	Asymptotic variance of the weighted mean.

`Theta.weibull`	Preliminary computations for the IF-s.
`IF.tcmean.w`	IF of the truncated mean.
`IF.wgmean.w`	IF of the weighted mean.
`Theta.smw`	Preliminary computations for the IF-s of smoothed estimates.
`IF.tcmean.smw`	IF of the smoothed version of the truncated mean.

Gamma case

`D.E.gamma`	LD_D-estimates of α, σ, and μ.
`TD.gamma`	Truncated mean.
`WD.gamma`	Weighted mean.
`ATR.gamma`	Approximate quartile based truncated mean.
`AV.DE.gamma`	Asymptotic variance of the LD_D-estimate.
`AV.TD.gamma`	Asymptotic variance of the truncated mean.
`ACV.DE.gamma`	Asymptotic covriance of the LD_D-estimates.
`ACV.TD.gamma`	Asymptotic covariance of the TD-estimates.
`AV.WD.gamma`	Asymptotic variance of the weighted mean.
`Theta.gamma`	Preliminary computations for the IF-s.
`IF.tcmean.g`	IF of the truncated mean.
`IF.wgmean.g`	IF of the weighted mean.
`Theta.smg`	Preliminary computations for the IF-s of smoothed estimates.
`IF.tcmean.smg`	IF of the smoothed version of the truncated mean.

Lognormal case

`D.E.lnorm`	LD_D-estimates of α, σ, and μ.
`S.E.lnorm`	S-estimates of α and σ.
`TD.lnorm`	Truncated mean.
`TS.lnorm`	Truncated mean with S-estimates as initial values.
`WD.lnorm`	Weighted mean.
`ATR.lnorm`	Approximate quartile based truncated mean.
`AV.DE.lnorm`	Asymptotic variance of the LD_D-estimate.
`AV.TD.lnorm`	Asymptotic variance of the truncated mean.
`ACV.DE.lnorm`	Asymptotic covariance of the LD_D-estimate.
`ACV.TD.lnorm`	Asymptotic covariance of the truncated mean.
`ACV.TS.lnorm`	Asymptotic covariance of the truncated mean with initial S-estimate.
`AV.WD.lnorm`	Asymptotic variance of the weighted mean.
`Theta.lnorm`	Preliminary computations for the IF-s.
`IF.tcmean.l`	IF of the truncated mean.
`IF.Sestim.l`	IF of the truncated mean with initial S-estimates.
`IF.wgmean.l`	IF of the weighted mean.
`Theta.sml`	Preliminary computations for the IF-s of smoothed estimates.
`IF.tcmean.sml`	IF of the smoothed version of the truncated mean.

Control parameters

`Cntrlpar`	Control parameters.

The algorithms are partially programmed in FORTRAN using the subroutine library ROBETH (Marazzi, 1993). They must be compiled and loaded into S-plus. A library is made available for S-plus for Windows; it can be loaded using the command `library(twmean,T)`.

function **D.E.weibull**

LD_D-estimates of the parameters of the Weibull distribution.

Specification

```
function D.E.weibull(x, control=Cntrlpar(...),...)
```

Purpose

This function computes the estimate $\tilde{\theta} = (\tilde{\alpha}, \tilde{\sigma})$ defined in MR99 (Section 2.1) for the location and scale parameters of the distribution of $\ln(X)$. It returns the estimates $\tilde{\vartheta}_1 = 1/\tilde{\sigma}$ and $\tilde{\vartheta}_2 = \exp(\tilde{\alpha})$ of the shape and scale parameters of the distribution of X, as well as $\tilde{\mu} = \tilde{\vartheta}_2 \Gamma(1 + 1/\tilde{\vartheta}_1)$. Moreover, it returns $m(F_n)$ and $s(F_n)$ according to (2.1.5)-(2.1.6) in MR99, as well as an estimate of the asymptotic variance of $\tilde{\mu}$.

Method

The transformation $Y = \ln(X)$ is used, the measures $m(F_n)$ and $s(F_n)$ are computed according to (2.1.5)-(2.1.6) in MR99, and equation (2.1.1) in MR99 is solved for $\tilde{\alpha}$ and $\tilde{\sigma}$. The shape and scale parameters of the Weibull distribution are estimated by $\tilde{\vartheta}_1 = 1/\tilde{\sigma}$ and $\tilde{\vartheta}_2 = \exp(\tilde{\alpha})$. The asymptotic variance of $\tilde{\mu}$ is computed using `AV.DE.weibull`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`shape`	Estimate $\tilde{\vartheta}_1 = 1/\tilde{\sigma}$.
`scale`	Estimate $\tilde{\vartheta}_2 = \exp(\tilde{\alpha})$.
`mu`	Estimate $\tilde{\mu} = \tilde{\vartheta}_2 \Gamma(1 + 1/\tilde{\vartheta}_1)$.
`m`	$m(F_n)$.
`s`	$s(F_n)$.
`V.mu`	Asymptotic variance of $\tilde{\mu}$.
`call`	The calling sequence.

function **`TD.weibull`**

Truncated mean estimate for the Weibull distribution.

Specification

```
function TD.weibull(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes T_u, T_l and the truncated mean estimate $\check{\mu}$ defined in MR99. The LD_D-estimates of shape and scale, the values of T_l and T_u, as well as an estimate of the asymptotic variance of $\check{\mu}$ are returned.

Method

The initial LD_D-estimates of shape and scale are computed by means of `D.E.weibull`. T_l is computed using a regula falsi procedure for solving (2.2.1) in MR99. The asymptotic variance of $\check{\mu}$ is estimated with the help of `AV.TD.weibull`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`mu`	Estimate $\check{\mu}$.
`shape`	Estimate $\tilde{\vartheta}_1 = 1/\tilde{\sigma}$ (returned by `D.E.weibull`).
`scale`	Estimate $\tilde{\vartheta}_2 = \exp(\tilde{\alpha})$ (returned by `D.E.weibull`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`V.mu`	Asymptotic variance of $\check{\mu}$.
`call`	The calling sequence.

function **WD.weibull**

Weighted mean estimate for the Weibull distribution.

Specification

```
function WD.weibull(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes T_u, T_l and the weighted mean estimate $\breve{\mu}$ defined in MR99. The LD_D-estimates of shape and scale, the values of T_l and T_u, as well as an estimate of the asymptotic variance of $\breve{\mu}$ are returned.

Method

The initial LD_D-estimates of shape and scale are computed by means of `D.E.weibull`. T_l is computed using of a regula falsi procedure. The asymptotic variance of $\breve{\mu}$ is estimated with the help of `AV.WD.weibull`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`mu`	Estimate $\breve{\mu}$.
`shape`	Estimate $\tilde{\vartheta}_1 = 1/\tilde{\sigma}$ (returned by `D.E.weibull`).
`scale`	Estimate $\tilde{\vartheta}_2 = \exp(\tilde{\alpha})$ (returned by `D.E.weibull`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`num`	Value of $\int x w(x, T_l, T_u) dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$.
`den`	Value of $\int w(x, T_l, T_u) dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`V.mu`	Asymptotic variance of $\breve{\mu}$.
`call`	The calling sequence.

function **ATR.weibull**

Approximate truncated mean estimate for the Weibull distribution.

Specification

```
function ATR.weibull(x)
```

Purpose

This function computes T_u, T_l and the approximate quartile based truncated mean $\hat{\mu}$ defined in MR98. The values of T_l and T_u, as well as an estimate of the asymptotic variance of $\hat{\mu}$ are returned.

Method

See MR98.

Arguments

`x`	Observation vector.

Value

List with the following components

`mu`	Estimate $\hat{\mu}$.
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`V.mu`	Asymptotic variance of $\hat{\mu}$.
`ok`	`ok` $= 0$ on output, indicates that the approximation is not satisfactory. Another estimate should be used.
`call`	The calling sequence.

function **AV.DE.weibull**

Asymptotic variance of the LD_D-estimate of μ: Weibull case.

Specification

```
function AV.DE.weibull(shape, scale, beta=0.4, gam=0.4)
```

Purpose

This function returns an estimate of the asymptotic variance of $\tilde{\mu} = \tilde{\vartheta}_2 \Gamma(1 + 1/\tilde{\vartheta}_1)$, where ϑ_1 and ϑ_2 are the shape and scale parameters of the Weibull distribution.

Method

The asymptotic variance $v(\tilde{\mu}, G)$ is estimated by $\int IF^2(x, \tilde{\mu}, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$, according to MR99. The values of $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ must be supplied as input arguments.

Arguments

`shape`	Value of $\tilde{\vartheta}_1$.
`scale`	Value of $\tilde{\vartheta}_2$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

Estimated value of $v(\tilde{\mu}, G)$.

function **AV.TD.weibull**

Asymptotic variance of the truncated mean: Weibull case.

Specification

```
function AV.TD.weibull(shape, scale, u=0.99, beta=0.4, gam=0.4,
         est=0)
```

Purpose

This function returns an estimate of the asymptotic variance of estimate defined by the argument `est`. In particular, it returns an estimate of the asymptotic variance of the truncated mean $\breve{\mu}$.

Method

The asymptotic variance $v(\texttt{est}, G)$ is estimated by $\int IF^2(x, \texttt{est}, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$, according to MR99. Here, $IF(x, \texttt{est}, G)$ denotes the influence function of the estimate `est`. The values of $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ must be supplied as input arguments.

Arguments

`shape`	Value of $\tilde{\vartheta}_1$.
`scale`	Value of $\tilde{\vartheta}_2$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`est`	Option parameter defining the estimate. The default, `est` = 0, corresponds to the truncated mean estimate $\breve{\mu}$. The special case `est=-4` computes the asymptotic covariance of (α, σ). The option list is given in `IF.tcmean.w`.

Value

Estimated value of the asymptotic variance.

function **ACV.DE.weibull**

Asymptotic covariance of the LD_D-estimate of ϑ_1, ϑ_2 and μ: Weibull case.

Specification

```
function ACV.DE.weibull(x,shape, scale, beta=0.4, gam=0.4,
         cov=c("Expected","Empirical")
```

Purpose

This function returns an estimate of the asymptotic covariance of $\tilde{\vartheta}_1$, $\tilde{\vartheta}_2$ and $\tilde{\mu} = \tilde{\vartheta}_2\Gamma(1 + 1/\tilde{\vartheta}_1)$, where ϑ_1 and ϑ_2 are the shape and scale parameters of the Weibull distribution. Two options are available: "Expected" or "Empirical".

Method

The asymptotic covariance $cov(\tilde{\vartheta}_1, \tilde{\vartheta}_2, \tilde{\mu}, G)$ is estimated by $\int IF(x, (\tilde{\vartheta}_1, \tilde{\vartheta}_2, \tilde{\mu}), G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) * IF^T(x, (\tilde{\vartheta}_1, \tilde{\vartheta}_2, \tilde{\mu}), G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$, according to MR99. The values of $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ must be supplied as input arguments. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

`x`	Observation vector.
`shape`	Value of $\tilde{\vartheta}_1$.
`scale`	Value of $\tilde{\vartheta}_2$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`cov`	Option for the computation of the covariance matrix. See Method.

Value

Estimated value of asymptotic covariance.

function **ACV.TD.weibull**

Asymptotic covariance of ϑ_1, ϑ_2 and the truncated mean: Weibull case.

Specification

```
function ACV.TD.weibull(x, shape, scale, u=0.99, beta=0.4, gam=0.4,
         cov=c("Expected","Empirical"))
```

Purpose

This function returns an estimate of the asymptotic covariance of $\tilde{\vartheta}_1$, $\tilde{\vartheta}_2$ and the truncated mean $\check{\mu}$, where ϑ_1 and ϑ_2 are the shape and scale parameters of the Weibull distribution. Two options are available: "Expected" or "Empirical".

Method

The asymptotic covariance $cov(\tilde{\vartheta}_1, \tilde{\vartheta}_2, \check{\mu}, G)$ is estimated by $\int IF(x, (\tilde{\vartheta}_1, \tilde{\vartheta}_2, \check{\mu}), G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) * IF^T(x, (\tilde{\vartheta}_1, \tilde{\vartheta}_2, \check{\mu}), G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$, according to MR99. The values of $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ must be supplied as input arguments. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

`x`	Observation vector.
`shape`	Value of $\tilde{\vartheta}_1$.
`scale`	Value of $\tilde{\vartheta}_2$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`cov`	Option for the computation of the covariance matrix. See Method.

Value

Estimated value of the asymptotic covariance.

function **AV.WD.weibull**

Asymptotic variance of the weighted mean: Weibull case.

Specification

```
function AV.WD.weibull(shape, scale, u=0.99, beta=0.4, gam=0.4,
         est=0)
```

Purpose

This function returns an estimate of the asymptotic variance of the estimate defined by the argument `est`. In particular, it returns an estimate of the asymptotic variance of the weighted mean $\breve{\mu}$.

Method

The asymptotic variance $v(\texttt{est}, G)$ is estimated by $\int IF^2(x, \texttt{est}, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2})dG_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}(x)$, according to MR99. Here, $IF(x; \texttt{est}, G)$ denotes the influence function of the estimate `est`. The values of $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ must be supplied as input arguments.

Arguments

`shape`	Value of $\tilde{\vartheta}_1$.
`scale`	Value of $\tilde{\vartheta}_2$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`est`	Option parameter defining the estimate. The default, `est=0`, corresponds to the weighted mean estimate $\breve{\mu}$. The special case `est=-4` computes the asymptotic covariance of $(\vartheta_1, \vartheta_2)$. The option list is given in `IF.wgmean.w`.

Value

Estimated value of the asymptotic variance.

function **Theta.weibull**

Preliminary computations for influence functions: Weibull case.

Specification

```
function Theta.weibull(shape, scale, u=0.99, beta=0.4, gam=0.4)
```

Purpose

This function returns a list of values required by `IF.tcmean.w` and `IF.wgmean.w`.

Arguments

`shape`	Value of $\tilde{\vartheta}_1$.
`scale`	Value of $\tilde{\vartheta}_2$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

If `beta` < 0.5, a list of 60 values; otherwise a list of 35 values.

function **IF.tcmean.w**

Influence function of the truncated mean at the estimated model: Weibull case.

Specification

```
function IF.tcmean.w(x, Theta, est=0)
```

Purpose

This function returns $IF(x; \texttt{est}, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2})$, where `est` denotes an estimate related with the computation of the truncated mean estimate.

Method

See MR99.

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.weibull`.
`est`	Option parameter defining the estimate. `est` = 0: truncated mean. `est` = 1: $m(F)$. `est` = 2: $s(F)$. `est` = 3: LD_D-estimate of ϑ_1. `est` = 4: LD_D-estimate of ϑ_2. `est` = 5: LD_D-estimate of μ. `est` = 6: estimate of T_l. `est` = 7: estimate of T_u.

Value

A vector of same length as `x`. If `est` $= -4$ the vector is the product $IF(x; \vartheta_1, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) * IF(x; \vartheta_2, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2})$.

`function` **`IF.wgmean.w`**

Influence functions of the weighted mean at the estimated model: Weibull case.

Specification

```
function IF.wgmean.w(x, Theta, est=0)
```

Purpose

This function returns $IF(x; \texttt{est}, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2})$, where `est` denotes an estimate related with the computation of the weighted mean estimate.

Method

See MR99.

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.weibull`.
`est`	Option parameter defining the estimate. $\texttt{est} = 0$: weighted mean. $\texttt{est} = 1$: $m(F)$. $\texttt{est} = 2$: $s(F)$. $\texttt{est} = 3$: LD_D-estimate of ϑ_1. $\texttt{est} = 4$: LD_D-estimate of ϑ_2. $\texttt{est} = 5$: LD_D-estimate of μ. $\texttt{est} = 6$: estimate of T_l. $\texttt{est} = 7$: estimate of T_u.

Value

A vector of same length as `x`. If $\texttt{est} = -4$ the vector is the product $IF(x; \vartheta_1, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2}) * IF(x; \vartheta_2, G_{\tilde{\vartheta}_1,\tilde{\vartheta}_2})$.

function **Theta.smw**

Preliminary computations for influence functions of smoothed estimates: Weibull case.

Specification

```
function Theta.smw(x, p, delta, u=0.99, beta=0.4, gam=0.4)
```

Purpose

Returns a list of values required by `IF.tcmean.smw`.

Arguments

`x`	Vector of values defining the support of the discrete distribution F_p.
`p`	Vector of probabilities p_i such that $\sum_i p_i = 1$).
`delta`	Vector of (positive) standard deviations δ_i for the smooth.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

A list of 56 values.

function **IF.tcmean.smw**

Influence function of a smoothed version of the truncated mean $\check{\mu}$, when the underlying distribution is discrete: Weibull case.

Specification

```
function IF.tcmean.smw(xj, x, delta, est=1, Theta, limint=NA)
```

Purpose

This function returns $IF(x_j; \texttt{est}, \bar{F}_p)$ and $IF(x_j; \overline{\texttt{est}}, F_p)$, where $\overline{\texttt{est}}$ is a smoothed version of the truncated mean $\check{\mu}$ or the trimmed mean m and F_p is a discrete distribution with support $x_1, \ldots, x_n$. See MR99 and MA01.

Method

The estimator $\overline{\texttt{est}}$ is defined by $\overline{\texttt{est}}(F_p) = \texttt{est}(\bar{F}_p)$, where $\bar{F}_p = \sum_{i=1}^{n} p_i G_{x_i}^{\delta_i}$ and $G_{x_i}^{\delta_i}$ denotes the normal cumulative distribution function with location x_i and scale δ_i. Thus,

$$IF(x_j; \overline{\texttt{est}}, F_p) = \int IF(y; \texttt{est}, \bar{F}_p) dG_{x_j}^{\delta_j}(y).$$

Arguments

`xj`	Subset of $\{x_1, \ldots, x_n\}$: the IF-s are evaluated at x_j, for all $x_j \in$ `xj`.
`x`	Vector of values $x_1, \ldots, x_n$ with $x_i \neq x_j$ for $i \neq j$: support of F_p.
`delta`	Vector of the (positive) standard deviations δ_i.
`est`	If `est=1`, the truncated mean $\check{\mu}$ is the estimate of interest. If `est=2`, the trimmed mean m is the estimate of interest.
`Theta`	Result of a preliminary call to `Theta.smw`.
`limint`	Limits of integration. For each i, the limits of integration are $x_i -$ `limint` and $x_i +$ `limint`. The defaults are $x_i - 5\delta_i$ and $x_i + 5\delta_i$.

Value

Two vectors: `IF1` $= IF(x_j; \texttt{est}, \bar{F}_p)$ and `IF2` $= IF(x_j; \overline{\texttt{est}}, F_p)$.

`function` **`D.E.gamma`**

LD_D-estimates of the parameters of the Gamma distribution.

Specification

```
function D.E.gamma(x, control=Cntrlpar(...), ...)
```

Purpose

This function returns the LD_D-estimates $\tilde{\alpha}$ and $\tilde{\sigma}$ of the shape and scale parameter of the distribution of X, as well as $\tilde{\mu} = \tilde{\alpha}\tilde{\sigma}$. These estimates are defined in MR99. Moreover, it returns $m(F_n)$ and $s(F_n)$ (according to (2.1.5)-(2.1.6) in MR99), as well as an estimate of the asymptotic variance of $\tilde{\mu}$.

Method

The measures $m(F_n)$ and $s(F_n)$ are computed according to (2.1.5)-(2.1.6) in MR99, and equation (2.1.2) in MR99 is solved for $\tilde{\alpha}$ and $\tilde{\sigma}$. The asymptotic variance of $\tilde{\mu}$ is computed with the help of `AV.DE.gamma`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`alpha`	Estimate $\tilde{\alpha}$.
`sigma`	Estimate $\tilde{\sigma}$.
`mu`	Estimate $\tilde{\mu} = \tilde{\alpha}\tilde{\sigma}$.
`m`	$m(F_n)$.
`s`	$s(F_n)$.
`V.mu`	Asymptotic variance of $\tilde{\mu}$.
`call`	The calling sequence.

`function` **`TD.gamma`**

Truncated mean estimate for the Gamma distribution.

Specification

```
function TD.gamma(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes T_l and T_u and the truncated mean estimate $\breve{\mu}$ defined in MR99. The LD$_D$-estimates $\tilde{\alpha}$ and $\tilde{\sigma}$ of shape and scale, the values of T_l and T_u, as well as an estimate of the asymptotic variance of $\breve{\mu}$ are returned.

Method

The initial LD$_D$-estimate of shape and scale are computed with the help of `D.E.gamma`. T_l is computed using a regula falsi procedure for solving (2.2.1) in MR99. The asymptotic variance of $\breve{\mu}$ is estimated with the help of `AV.TD.gamma`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. Their default values are given by `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`mu`	Estimate $\breve{\mu}$.
`alpha`	Estimate $\tilde{\alpha}$ (returned by `D.E.gamma`).
`sigma`	Estimate $\tilde{\sigma}$ (returned by `D.E.gamma`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`V.mu`	Asymptotic variance of $\breve{\mu}$.
`call`	The calling sequence.

function **WD.gamma**

Weighted mean estimate for the Gamma distribution.

Specification

```
function WD.gamma(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes T_l, T_u and the weighted mean estimate $\check{\mu}$ defined in MR99. The LD_D-estimates of shape and scale, the values of T_l and T_u, as well as an estimate of the asymptotic variance of $\check{\mu}$ are returned.

Method

The initial LD_D-estimate of shape and scale is computed with the help of `D.E.gamma`. T_l is computed using a regula falsi procedure. The asymptotic variance of $\check{\mu}$ is estimated with the help of `AV.WD.gamma`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. Their default values are given by `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`mu`	Estimate $\check{\mu}$.
`alpha`	Estimate $\tilde{\alpha}$ (returned by `D.E.gamma`).
`sigma`	Estimate $\tilde{\sigma}$ (returned by `D.E.gamma`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`num`	Value of $\int x w(x, T_l, T_u) dG_{\tilde{\alpha},\tilde{\sigma}}(x)$.
`den`	Value of $\int w(x, T_l, T_u) dG_{\tilde{\alpha},\tilde{\sigma}}(x)$.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`V.mu`	Asymptotic variance of $\check{\mu}$.
`call`	The calling sequence.

function **ATR.gamma**

Approximate truncated mean estimate for the Gamma distribution.

Specification

```
function ATR.gamma(x)
```

Purpose

This function computes T_u, T_l and the approximate quartile based truncated mean estimate $\hat{\mu}$ defined in MR98. The values of T_l and T_u, as well as an estimate of the asymptotic variance of $\hat{\mu}$ are returned.

Method

See MR98.

Arguments

`x`	Observation vector.

Value

List with the following components

`mu`	Estimate $\hat{\mu}$.
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`V.mu`	Asymptotic variance of $\hat{\mu}$.
`ok`	`ok` $= 0$ on output, indicates that the approximation is not satisfactory. Another estimate should be used.
`call`	The calling sequence.

`function` **`AV.DE.gamma`**

Asymptotic variance of the LD_D-estimate of μ: Gamma case.

Specification

```
function AV.DE.gamma(alpha, sigma, beta=0.4, gam=0.4)
```

Purpose

This function returns an estimate of the asymptotic variance of $\tilde{\mu} = \tilde{\alpha}\tilde{\sigma}$ where α and σ are the shape and scale parameters of the gamma distribution.

Method

The asymptotic variance $v(\tilde{\mu}, G)$ is estimated by $\int IF^2(x, \tilde{\mu}, G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

Estimated value of $v(\tilde{\mu}, G)$.

function **AV.TD.gamma**

Asymptotic variance of the truncated mean: Gamma case.

Specification

```
function AV.TD.gamma(alpha, sigma, u=0.99, beta=0.4, gam=0.4,
         est=0)
```

Purpose

This function returns an estimate of the asymptotic variance of the estimate defined by the argument `est`. In particular, it returns an estimate of the asymptotic variance of the truncated mean $\breve{\mu}$.

Method

The asymptotic variance $v(\texttt{est}, G)$ is estimated by $\int IF^2(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. Here, $IF(x, \texttt{est}, G)$ denotes the influence function of the estimate `est`. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`est`	Option parameter defining the estimate. The default, `est` $= 0$, corresponds to the truncated mean estimate $\breve{\mu}$. The special case `est=-4` computes the asymptotic covariance of (α, σ). The option list is given in `IF.tcmean.g`.

Value

Estimated value of the asymptotic variance.

function **ACV.DE.gamma**

Asymptotic covariance of the LD_D-estimate of α, σ and μ: Gamma case.

Specification

```
function ACV.DE.gamma(x, alpha, sigma, beta=0.4, gam=0.4,
         cov=c("Expected","Empirical")
```

Purpose

This function returns an estimate of the asymptotic covariance of $\tilde{\alpha}$, $\tilde{\sigma}$ and $\tilde{\mu} = \tilde{\alpha}\tilde{\sigma}$, where α and σ are the shape and scale parameters of the gamma distribution. Two options are available: "Expected" or "Empirical".

Method

The asymptotic covariance $cov(\tilde{\alpha}, \tilde{\sigma}, \tilde{\mu}, G)$ is estimated by $\int IF(x, (\tilde{\alpha}, \tilde{\sigma}, \tilde{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) * IF^T(x, (\tilde{\alpha}, \tilde{\sigma}, \tilde{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

`x`	Observation vector.
`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`cov`	Option for the computation of the covariance matrix. See Method.

Value

Estimated value of asymptotic covariance.

function **ACV.TD.gamma**

Asymptotic covariance of α, σ and the truncated mean: Gamma case.

Specification

```
function ACV.TD.gamma(x, alpha, sigma, u=0.99, beta=0.4, gam=0.4,
         cov=c("Expected","Empirical"))
```

Purpose

This function returns an estimate of the asymptotic covariance of $\tilde{\alpha}$, $\tilde{\sigma}$ and the truncated mean $\check{\mu}$, where α and σ are the shape and scale parameters of the gamma distribution. Two options are available: "Expected" or "Empirical".

Method

The asymptotic covariance $cov(\tilde{\alpha}, \tilde{\sigma}, \check{\mu}, G)$ is estimated by $\int IF(x, (\tilde{\alpha}, \tilde{\sigma}, \check{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) * IF^T(x, (\tilde{\alpha}, \tilde{\sigma}, \check{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

`x`	Observation vector.
`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`cov`	Option for the computation of the covariance matrix. See Method.

Value

Estimated value of the asymptotic covariance.

`function` **`AV.WD.gamma`**

Asymptotic variance of the weighted mean: Gamma case.

Specification

```
function AV.WD.gamma(alpha, sigma, u=0.99, beta=0.4, gam=0.4,
         est=0)
```

Purpose

This function returns an estimate of the asymptotic variance of the estimate defined by the argument `est`. In particular, it returns an estimate of the asymptotic variance of the weighted mean $\breve{\mu}$.

Method

The asymptotic variance $v(\texttt{est}, G)$ is estimated by $\int IF^2(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. Here, $IF(x, \texttt{est}, G)$ denotes the influence function of the estimate `est`. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`est`	Option parameter defining the estimate. The default, `est` $= 0$, corresponds to the weighted mean estimate $\breve{\mu}$. The special case `est=-4` computes the asymptotic covariance of (α, σ). The option list is given in `IF.wgmean.g`.

Value

Estimated value of the asymptotic variance.

function **Theta.gamma**

Preliminary computations for the influence functions: Gamma case.

Specification

```
function Theta.gamma(alpha, sigma, u=0.99, beta=0.4, gam=0.4)
```

Purpose

This function returns a list of values required by `IF.tcmean.g` and `IF.wgmean.g`.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

If `beta` < 0.5, a list of 60 values; otherwise a list of 35 values.

`function` **`IF.tcmean.g`**

Influence function of the truncated mean at the estimated model: Gamma case.

Specification

```
function IF.tcmean.g(x, Theta, est=0)
```

Purpose

This function returns $IF(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})$, where `est` denotes an estimate related with the computation of the truncated mean estimate.

Method

See MR99.

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.gamma`.
`est`	Option parameter defining the estimate. `est` = 0: truncated mean. `est` = 1: $m(F)$. `est` = 2: $s(F)$. `est` = 3: LD_D-estimate of α. `est` = 4: LD_D-estimate of σ. `est` = 5: LD_D-estimate of μ. `est` = 6: estimate of T_l. `est` = 7, estimate of T_u.

Value

A vector of same length as `x`. If `est` $= -4$ the vector is the product $IF(x, \alpha, G_{\tilde{\alpha},\tilde{\sigma}}) * IF(x, \sigma, G_{\tilde{\alpha},\tilde{\sigma}})$.

function **IF.wgmean.g**

Influence functions of the weighted mean at the estimated model: Gamma case.

Specification

```
function IF.wgmean.g(x, Theta, est=0)
```

Purpose

This function returns $IF(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})$, where `est` denotes an estimate related with the computation of the weighted mean estimate.

Method

See MR99.

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.gamma`.
`est`	Option parameter defining the estimate. $\texttt{est} = 0$: weighted mean. $\texttt{est} = 1$: $m(F)$. $\texttt{est} = 2$: $s(F)$. $\texttt{est} = 3$: LD_D-estimate of α. $\texttt{est} = 4$: LD_D-estimate of σ. $\texttt{est} = 5$: LD_D-estimate of μ. $\texttt{est} = 6$: estimate of T_l. $\texttt{est} = 7$, estimate of T_u.

Value

A vector of same length as `x`. If $\texttt{est} = -4$ the vector is the product $IF(x, \alpha, G_{\tilde{\alpha},\tilde{\sigma}}) * IF(x, \sigma, G_{\tilde{\alpha},\tilde{\sigma}})$.

function **Theta.smg**

Preliminary computations for influence functions of smoothed estimates: Gamma case.

Specification

```
function Theta.smg(x, p, delta, u=0.99, beta=0.4, gam=0.4)
```

Purpose

Returns a list of values required by `IF.tcmean.smg`.

Arguments

`x`	Vector of values defining the support of the discrete distribution F_p.
`p`	Vector of probabilities p_i such that $\sum_i p_i = 1$).
`delta`	Vector of (positive) standard deviations δ_i for the smooth.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

A list of 56 values.

function IF.tcmean.smg

Influence function of a smoothed version of the truncated mean $\check{\mu}$, when the underlying distribution is discrete: Gamma case.

Specification

```
function IF.tcmean.smg(xj, x, delta, est=1, Theta, limint=NA)
```

Purpose

This function returns $IF(x_j; \texttt{est}, \bar{F}_p)$ and $IF(x_j; \overline{\texttt{est}}, F_p)$, where $\overline{\texttt{est}}$ is a smoothed version of the truncated mean $\check{\mu}$ or the trimmed mean m and F_p is a discrete distribution with support $x_1, \ldots, x_n$. See MR99 and MA01.

Method

The estimator $\overline{\texttt{est}}$ is defined by $\overline{\texttt{est}}(F_p) = \texttt{est}(\bar{F}_p)$, where $\bar{F}_p = \sum_{i=1}^n p_i G_{x_i}^{\delta_i}$ and $G_{x_i}^{\delta_i}$ denotes the normal cumulative distribution function with location x_i and scale δ_i. Thus,

$$IF(x_j; \overline{\texttt{est}}, F_p) = \int IF(y; \texttt{est}, \bar{F}_p) dG_{x_j}^{\delta_j}(y).$$

Arguments

`xj`	Subset of $\{x_1, \ldots, x_n\}$: the IF-s are evaluated at x_j, for all $x_j \in$ `xj`.
`x`	Vector of values $x_1, \ldots, x_n$ with $x_i \neq x_j$ for $i \neq j$: support of F_p.
`delta`	Vector of the (positive) standard deviations δ_i.
`est`	If `est=1`, the truncated mean $\check{\mu}$ is the estimate of interest.
	If `est=2`, the trimmed mean m is the estimate of interest.
`Theta`	Result of a preliminary call to `Theta.smg`.
`limint`	Limits of integration. For each i, the limits of integration are $x_i -$ `limint` and $x_i +$ `limint`. The defaults are $x_i - 5\delta_i$ and $x_i + 5\delta_i$.

Value

Two vectors: `IF1` $= IF(x_j; \texttt{est}, \bar{F}_p)$ and `IF2` $= IF(x_j; \overline{\texttt{est}}, F_p)$.

function **`D.E.lnorm`**

LD_D-estimates of the parameters of the Lognormal distribution.

Specification

```
function D.E.lnorm(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes the LD_D-estimate $\tilde{\theta} = (\tilde{\alpha}, \tilde{\sigma})$, defined in MR99 (Section 2.1) for the location and scale parameters of the Gaussian distribution of $\ln(X)$, as well as $\tilde{\mu} = \exp(\tilde{\alpha} + \tilde{\sigma}^2/2)$. Moreover, it returns $m(F_n)$ and $s(F_n)$ according to (2.1.5)-(2.1.6) in MR99, as well as an estimate of the asymptotic variance of $\tilde{\mu}$.

Method

The transformation $Y = \ln(X)$ is used, the measures $m(F_n)$ and $s(F_n)$ are computed according to (2.1.5)-(2.1.6) in MR99, and equation (2.1.1) in MR99 is solved for $\tilde{\alpha}$ and $\tilde{\sigma}$. The asymptotic variance of $\tilde{\mu}$ is estimated using `AV.DE.lnorm`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`alpha`	Estimate $\tilde{\alpha}$.
`sigma`	Estimate $\tilde{\sigma}$.
`mu`	Estimate $\tilde{\mu} = \exp(\tilde{\alpha} + 0.5\tilde{\sigma}^2)$.
`m`	$m(F_n)$.
`s`	$s(F_n)$.
`V.mu`	Asymptotic variance of $\tilde{\mu}$.
`call`	The calling sequence.

function **S.E.lnorm**

S-estimates of the parameters of the Lognormal distribution.

Specification

```
function S.E.lnorm(x, cov=c("None","Expected","Empirical"),
         control=MM.E.control(...), ...)
```

Purpose

This function computes the S-estimate $\tilde{\theta} = (\tilde{\alpha}, \tilde{\sigma})$, defined in Marazzi [1997] (function MM-E.gauss) for the location and scale parameters of the Gaussian distribution of $\ln(X)$. Moreover, it returns an estimate of the asymptotic covariance of $(\tilde{\alpha}, \tilde{\sigma})$.

Method

The transformation $Y = \ln(X)$ is used and the function `MM.E.gauss` from the `robnrm` library is applied. If `(COV` $\neq$ `"None")`, the asymptotic covariance $cov(\tilde{\alpha}, \tilde{\sigma}, G)$ is estimated by $\int IF(x,(\tilde{\alpha},\tilde{\sigma}),G_{\tilde{\alpha},\tilde{\sigma}}) * IF^T(x,(\tilde{\alpha},\tilde{\sigma}),G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, where `IF` is given by the function `IF.Sestim.l`. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

`x`	Observation vector.
`cov`	Option for the computation of the covariance matrix. See Method.
`control`	List including all the control parameters. The default values are set with the help of `MM.E.control`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`alpha`	Estimate $\tilde{\alpha}$.
`sigma`	Estimate $\tilde{\sigma}$.
`COV`	Asymptotic covariance of $(\tilde{\alpha}, \tilde{\sigma})$.
`call`	The calling sequence.

`function` **`TD.lnorm`**

Truncated mean estimate for the Lognormal distribution.

Specification

```
function TD.lnorm(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes T_u and T_l and the truncated mean estimate $\breve{\mu}$ defined in MR99. The LD_D-estimate of location and scale, the values of T_l and T_u, as well as an estimate of the asymptotic variance of $\breve{\mu}$ are returned.

Method

The initial LD_D-estimate of location and scale are computed with the help of `D.E.lnorm`. T_l is computed using a regula falsi procedure for solving (2.2.1) in MR99. The asymptotic variance of $\breve{\mu}$ is estimated with the help of `AV.TD.lnorm`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`mu`	Estimate $\breve{\mu}$.
`alpha`	Estimate $\tilde{\alpha}$ (returned by `D.E.lnorm`).
`sigma`	Estimate $\tilde{\sigma}$ (returned by `D.E.lnorm`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`V.mu`	Asymptotic variance of $\breve{\mu}$.
`call`	The calling sequence.

function **TS.lnorm**

Truncated mean with initial S-estimates for the Lognormal distribution.

Specification

```
function TS.lnorm(x, u, ctrl.S,cov=c("None","Expected","Empirical"))
```

Purpose

This function computes T_u and T_l and the truncated mean estimate $\check{\mu}$ defined in MR99. The S-estimates of location and scale, the values of T_l and T_u, as well as an estimate of the asymptotic covariance of $(\tilde{\alpha}, \tilde{\sigma}, \check{\mu})$ are returned.

Method

The initial S-estimate of location and scale are computed with the help of `S.E.lnorm`. T_l is computed using a regula falsi procedure for solving (2.2.1) in MR99. The asymptotic covariance of $(\tilde{\alpha}, \tilde{\sigma}, \check{\mu})$ is estimated with the help of `ACV.TD.lnorm`.

Arguments

`x`	Observation vector.
`u`	Value of the tuning constant u.
`ctrl.S`	List including all the control parameters. The default values are set with the help of `MM.E.control`.
`cov`	Option for the computation of the asymptotic covariance matrix. See `ACV.TS.lnorm`.

Value

List with the following components

`mu`	Estimate $\check{\mu}$.
`alpha`	Estimate $\tilde{\alpha}$ (returned by `S.E.lnorm`).
`sigma`	Estimate $\tilde{\sigma}$ (returned by `S.E.lnorm`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`COV`	Asymptotic covariance of $(\tilde{\alpha}, \tilde{\sigma}, \check{\mu})$.
`call`	The calling sequence.

function **`WD.lnorm`**
Weighted mean estimate for the Lognormal distribution.

Specification

```
function WD.lnorm(x, control=Cntrlpar(...), ...)
```

Purpose

This function computes T_u and T_l and the weighted mean estimate $\check{\mu}$ defined in MR99. The LD$_D$-estimate of location and scale, the values of T_l and T_u, as well as an estimate of the asymptotic variance of $\check{\mu}$ are returned.

Method

The initial LD$_D$-estimate of location and scale are computed with the help of `D.E.lnorm`. T_l is computed using a regula falsi procedure. The asymptotic variance of $\check{\mu}$ is estimated with the help of `AV.TD.lnorm`.

Arguments

`x`	Observation vector.
`control`	List including all the control parameters. The default values are set with the help of `Cntrlpar`.
`...`	List of the control parameters to be changed.

Value

List with the following components

`mu`	Estimate $\check{\mu}$.
`alpha`	Estimate $\tilde{\alpha}$ (returned by `D.E.lnorm`).
`sigma`	Estimate $\tilde{\sigma}$ (returned by `D.E.lnorm`).
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`num`	Value of $\int xw(x, T_l, T_u)dG_{\tilde{\alpha},\tilde{\sigma}}(x)$.
`den`	Value of $\int w(x, T_l, T_u)dG_{\tilde{\alpha},\tilde{\sigma}}(x)$.
`ok`	`ok` $= 1$ on output, indicates that a solution has been reached within the desired accuracy.
`V.mu`	Asymptotic variance of $\check{\mu}$.
`call`	The calling sequence.

function **`ATR.lnorm`**

Approximate truncated mean estimate for the Lognormal distribution.

Specification

```
function ATR.lnorm(x)
```

Purpose

This function computes T_u, T_l and the approximate quartile based truncated mean estimate $\hat{\mu}$ defined in MR98. The values of T_l and T_u, as well as an estimate of the asymptotic variance of $\hat{\mu}$ are returned.

Method

See MR98.

Arguments

`x`	Observation vector.

Value

List with the following components

`mu`	Estimate $\hat{\mu}$.
`Tl`	Value of T_l.
`Tu`	Value of T_u.
`V.mu`	Asymptotic variance of $\hat{\mu}$.
`ok`	`ok` = 0 on output, indicates that the approximation is not satisfactory. Another estimate should be used.
`call`	The calling sequence.

`function` **`AV.DE.lnorm`**

Asymptotic variance of the LD_D-estimate of μ: Lognormal case.

Specification

```
function AV.DE.lnorm(alpha, sigma, beta=0.4, gam=0.4)
```

Purpose

This function returns an estimate of the asymptotic variance of the LD_D-estimate $\tilde{\mu} = \exp(\tilde{\alpha} + \tilde{\sigma}^2/2)$, where α and σ are the location and scale parameters of the Lognormal distribution.

Method

The asymptotic variance $v(\tilde{\mu}, G)$ is estimated by $\int IF^2(x, \tilde{\mu}, G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

Estimated value of $v(\tilde{\mu}, G)$.

function **AV.TD.lnorm**

Asymptotic variance of the truncated mean: Lognormal case.

Specification

```
function AV.TD.lnorm(alpha, sigma, u=0.4, beta=0.99, gam=0.99,
         est=0)
```

Purpose

This function returns an estimate of the asymptotic variance of the estimate define by the argument `est`. In particular, it returns an estimate of the asymptotic variance of the truncated mean estimate $\check{\mu}$.

Method

The asymptotic variance $v(\texttt{est}, G)$ is estimated by $\int IF^2(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. Here, $IF(x, \texttt{est}, G)$ denotes the influence function of the estimate defined by `est`. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`est`	Option parameter defining the estimate. The default, `est` $= 0$, corresponds to the truncated mean estimate $\check{\mu}$. The special case `est=-4` computes the asymptotic covariance of (α, σ). The option list is given in `IF.tcmean.l`.

Value

Estimated value of the asymptotic variance.

function `ACV.DE.lnorm`

Asymptotic covariance of the LD_D-estimate of α, σ and μ: Lognormal case.

Specification

```
function ACV.DE.lnorm(x,alpha, sigma, beta=0.4, gam=0.4,
         cov=c("Expected","Empirical")
```

Purpose

This function returns an estimate of the asymptotic covariance of $\tilde{\alpha}$, $\tilde{\sigma}$ and $\tilde{\mu} = \exp(\tilde{\alpha} + \tilde{\sigma}^2/2)$, where α and σ are the shape and scale parameters of the lognormal distribution. Two options are available: "Expected" or "Empirical".

Method

The asymptotic covariance $cov(\tilde{\alpha}, \tilde{\sigma}, \tilde{\mu}, G)$ is estimated by $\int IF(x, (\tilde{\alpha}, \tilde{\sigma}, \tilde{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) * IF^T (x, (\tilde{\alpha}, \tilde{\sigma}, \tilde{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

`x`	Observation vector.
`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`cov`	Option for the computation of the covariance matrix. See Method.

Value

Estimated value of asymptotic covariance.

function **ACV.TD.lnorm**

Asymptotic covariance of α, σ and the truncated mean: Lognormal case.

Specification

```
function ACV.TD.lnorm(x, alpha, sigma, u=0.99, beta=0.4, gam=0.4,
         cov=c("Expected","Empirical"))
```

Purpose

This function returns an estimate of the asymptotic covariance of $\tilde{\alpha}$, $\tilde{\sigma}$ and the truncated mean $\breve{\mu}$, where α and σ are the mean and standard deviation parameters of the lognormal distribution. Two options are available: "Expected" or "Empirical".

Method

The asymptotic covariance $cov(\tilde{\alpha}, \tilde{\sigma}, \breve{\mu}, G)$ is estimated by $\int IF(x, (\tilde{\alpha}, \tilde{\sigma}, \breve{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) * IF^T (x, (\tilde{\alpha}, \tilde{\sigma}, \breve{\mu}), G_{\tilde{\alpha},\tilde{\sigma}}) dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments. If option "Empirical" is choosen, the integrals are replaced by mean over the observations given as input argument.

Arguments

x	Observation vector.
alpha	Value of $\tilde{\alpha}$.
sigma	Value of $\tilde{\sigma}$.
u	Value of the parameter $u \in [0.5, 1]$.
beta	Value of β, $\beta \in [0, 0.5]$.
gam	Value of γ, $\gamma \in [0, 0.5]$.
cov	Option for the computation of the covariance matrix. See Method.

Value

Estimated value of the asymptotic covariance.

function **AV.WD.lnorm**

Asymptotic variance of the weighted mean: Lognormal case.

Specification

```
function AV.WD.lnorm(alpha, sigma, u=0.99, beta=0.4, gam=0.4,
         est=0)
```

Purpose

This function returns an estimate of the asymptotic variance of the estimate define by the argument `est`. In particular, it returns an estimate of the asymptotic variance of the weighted mean estimate $\breve{\mu}$.

Method

The asymptotic variance $v(\texttt{est}, G)$ is estimated by $\int IF^2(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})dG_{\tilde{\alpha},\tilde{\sigma}}(x)$, according to MR99. Here, $IF(x, \texttt{est}, G)$ denotes the influence function of the estimate defined by `est`. The values of $\tilde{\alpha}$ and $\tilde{\sigma}$ must be supplied as input arguments.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`est`	Option parameter defining the estimate. The default, `est` $= 0$, corresponds to the weighted mean estimate $\breve{\mu}$. The special case `est=-4` computes the asymptotic covariance of (α, σ). The option list is given in `IF.tcmean.l`.

Value

Estimated value of the asymptotic variance.

function **Theta.lnorm**

Preliminary computations for influence functions: Lognormal case.

Specification

```
function Theta.lnorm(alpha, sigma, u=0.99, beta=0.4, gam=0.4)
```

Purpose

This function returns a list of values required by `IF.tcmean.l` and `IF.wgmean.l`.

Arguments

`alpha`	Value of $\tilde{\alpha}$.
`sigma`	Value of $\tilde{\sigma}$.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5)$. A value of β equal to 0.5 is treated as $\beta = 0.4999$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

A list of 60 values.

`function` **`IF.tcmean.l`**

Influence function of the truncated mean at the estimated model: Lognormal case.

Specification

```
function IF.tcmean.l(x, Theta, est=0)
```

Purpose

This function returns $IF(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})$, where `est` denotes an estimate related with the computation of the truncated mean estimate.

Method

See MR99.

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.lnorm`.
`est`	Option parameter defining the estimate. `est` = 0: truncated mean. `est` = 1: $m(F)$. `est` = 2: $s(F)$. `est` = 3: LD_D-estimate of α. `est` = 4: LD_D-estimate of σ. `est` = 5: LD_D-estimate of μ. `est` = 6: estimate of T_l. `est` = 7, estimate of T_u.

Value

A vector of same length as `x`. If `est` $= -4$ the vector is the product $IF(x, \alpha, G_{\tilde{\alpha},\tilde{\sigma}}) * IF(x, \sigma, G_{\tilde{\alpha},\tilde{\sigma}})$.

function IF.Sestim.l

Influence function of the truncated mean with initial S-estimates at the estimated model: Lognormal case.

Specification

```
function IF.Sestim.l(x, Theta, est=0)
```

Purpose

This function returns $IF(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})$, where `est` denotes an estimate related with the computation of the truncated mean with initial S-estimate.

Method

See MR99?

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.lnorm`.
`est`	Option parameter defining the estimate. `est` $= 0$: truncated mean. `est` $= 3$: S-estimate of α. `est` $= 4$: S-estimate of σ. `est` $= 5$: S-estimate of $\mu = \exp(\alpha + \sigma^2/2)$. `est` $= 6$: estimate of T_l. `est` $= 7$, estimate of T_u.

Value

A vector of same length as `x`. If `est` $= -4$ the vector is the product $IF(x, \alpha, G_{\tilde{\alpha},\tilde{\sigma}}) * IF(x, \sigma, G_{\tilde{\alpha},\tilde{\sigma}})$.

function **IF.wgmean.l**

Influence function of the weighted mean at the estimated model: Lognormal case.

Specification

```
function IF.wgmean.l(x, Theta, est=0)
```

Purpose

This function returns $IF(x, \texttt{est}, G_{\tilde{\alpha},\tilde{\sigma}})$, where `est` denotes an estimate related with the computation of the weighted mean estimate.

Method

See MR99.

Arguments

`x`	Vector of observations.
`Theta`	Result of a preliminary call to `Theta.lnorm`.
`est`	Option parameter defining the estimate. `est` $= 0$: weighted mean. `est` $= 1$: $m(F)$. `est` $= 2$: $s(F)$. `est` $= 3$: LD$_D$-estimate of α. `est` $= 4$: LD$_D$-estimate of σ. `est` $= 5$: LD$_D$-estimate of μ. `est` $= 6$: estimate of T_l. `est` $= 7$, estimate of T_u.

Value

A vector of same length as `x`. If `est` $= -4$ the vector is the product $IF(x, \alpha, G_{\tilde{\alpha},\tilde{\sigma}}) * IF(x, \sigma, G_{\tilde{\alpha},\tilde{\sigma}})$.

function **Theta.sml**

Preliminary computations for influence functions of smoothed estimates: Lognormal case.

Specification

```
function Theta.sml(x, p, delta, u=0.99, beta=0.4, gam=0.4)
```

Purpose

Returns a list of values required by `IF.tcmean.sml`.

Arguments

`x`	Vector of values defining the support of the discrete distribution F_p.
`p`	Vector of probabilities p_i such that $\sum_i p_i = 1$).
`delta`	Vector of the (positive) standard deviations δ_i for the smooth.
`u`	Value of the parameter $u \in [0.5, 1]$.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.

Value

A list of 60 values.

function **IF.tcmean.sml**

Influence function of a smoothed version of the truncated mean $\breve{\mu}$, when the underlying distribution is discrete: Lognormal case.

Specification

```
function IF.tcmean.sml(xj, x, delta, est=1, Theta, limint=NA)
```

Purpose

This function returns $IF(x_j; \texttt{est}, \bar{F}_p)$ and $IF(x_j; \overline{\texttt{est}}, F_p)$, where $\overline{\texttt{est}}$ is a smoothed version of the truncated mean $\breve{\mu}$ or the trimmed mean m and F_p is a discrete distribution with support $x_1, \ldots, x_n$. See MR99 and MA01.

Method

The estimator $\overline{\texttt{est}}$ is defined by $\overline{\texttt{est}}(F_p) = \texttt{est}(\bar{F}_p)$, where $\bar{F}_p = \sum_{i=1}^n p_i G_{x_i}^{\delta_i}$ and $G_{x_i}^{\delta_i}$ denotes the normal cumulative distribution function with location x_i and scale δ_i. Thus,

$$IF(x_j; \overline{\texttt{est}}, F_p) = \int IF(y; \texttt{est}, \bar{F}_p) dG_{x_j}^{\delta_j}(y).$$

Arguments

`xj`	Subset of $\{x_1, \ldots, x_n\}$: the IF-s are evaluated at x_j, for all $x_j \in$ `xj`.
`x`	Vector of values $x_1, \ldots, x_n$ with $x_i \neq x_j$ for $i \neq j$: support of F_p.
`delta`	Vector of the (positive) standard deviations δ_i.
`est`	If `est=1`, the truncated mean $\breve{\mu}$ is the estimate of interest. If `est=2`, the trimmed mean m is the estimate of interest.
`Theta`	Result of a preliminary call to `Theta.sml`.
`limint`	Limits of integration. For each i, the limits of integration are $x_i -$ `limint` and $x_i +$ `limint`. The defaults are $x_i - 5\delta_i$ and $x_i + 5\delta_i$.

Value

Two vectors: `IF1` $= IF(x_j; \texttt{est}, \bar{F}_p)$ and `IF2` $= IF(x_j; \overline{\texttt{est}}, F_p)$.

`function` `Cntrlpar`

Control parameters for the functions described in this manual.

Specification

```
function Cntrlpar(alpha1=0.5, alpha2=20.5, u=0.99, beta=0.4, gam=0.4,
                  cov=F,tol=0.0001)
```

Purpose

This function returns a list of control parameters for the functions described in this manual (in particular, for the algorithms used in `D.E.gamma`, `TD.gamma` and `WD.gamma`).

Method

Direct assignement.

Arguments

`alpha1`	Minimum guess α_1 of $\tilde{\alpha}$ for `D.E.gamma`, `TD.gamma` and `WD.gamma`. The regula falsi procedure for solving (2.1.2) in MR99 is applied to the interval $[\alpha_1, \alpha_2]$.
`alpha2`	Maximum guess α_2 of $\tilde{\alpha}$ `D.E.gamma`, `TD.gamma` and `WD.gamma`. The regula falsi procedure for solving (2.1.2) in MR99 is applied to the interval $[\alpha_1, \alpha_2]$.
`u`	Value of the tuning constant u.
`beta`	Value of β, $\beta \in [0, 0.5]$.
`gam`	Value of γ, $\gamma \in [0, 0.5]$.
`cov`	If `cov=T`, estimate of the variance of $\check{\mu}$ is returned.
`tol`	Required relative precision of α, σ and T_l.

Value

List with the same components as in Arguments.

Examples

```
> library(robeth,T); library(robwbl,T); library(twmean,T)

# LD-D estimator with simulated data
>
> alpha <- 5; sigma <- 1; n  <- 200; eps <- 0.06
> .Random.seed <- c(61,52,34,16,34, 1,24,14, 7,52, 9, 1)
> n2 <- rbinom(1,size=n,prob=eps); n1 <- n-n2
> y  <- c(rgamma(n1,shape=alpha)*sigma, runif(n2,min=0,max=50))
> dy <- D.E.gamma(y,alpha2=10.5,cov=T)
> dy$alpha; dy$sigma               # [1] 4.738349  [1] 1.066627
> dy$mu; dy$V.mu                   # [1] 5.054052  [1] 7.923298

# TD-estimator with data of MR98, Table 3
>
> y  <-c(rep(1,2),rep(2,6),rep(3,5),rep(4,5),rep(5,4),rep(6,2),
        7,7,8,9,16,115,198,354)
> n <- length(y)
> dy <- TD.weibull(y,cov=T);
> dy$mu; dy$V.mu/n                 # [1] 4  [1] 0.3244401

# Asymptotic relative efficiency of TD- w.r.t ML-estimator
>
> vml <- AV.ML.weibull(dy$shape,dy$scale)$V.mu
> vtd <- AV.TD.weibull(dy$shape,dy$scale,u=0.99,beta=0.4,gam=0.4)
> are <- vml/vtd                   # [1] 0.79629 or simply
> are <- ARE.TD.weibull(dy$shape,u=0.99,beta=0.4,gam=0.4)

# Influence function of the smoothed truncated mean
>
> pyi   <- rep(1/n,n)
> tmp   <- Unique(y,pyi) # see below
> yu    <- tmp$yu; pu <- tmp$pu; yfrq <- tmp$yfrq
> delta <- rep(1,length(pu))
> Thety <- Theta.smg(yu,pu,delta,u=0.99,beta=0.4,gam=0.4)
> IF    <- IF.tcmean.smg(yu,yu,delta,est=1,Theta=Thety)
> IFmu  <- IF$IF1; IFsm <- IF$IF2
> win.graph()
> plot(yu,IFmu,type="n"); points(yu,IFmu,col=2)
> lines(yu, IFsm,col=3)
>
# Check that IFsm is a set of influences at a discrete distribution
> sum(pu*IFsm) # 4.091327e-06
```

The following function `Unique` reduces the vector `y` to a vector `yu` with different (unique) components and computes vectors `yfrq` and `pu` of absolute and relative frequencies.

```
> Unique <- function(yi,pi,ysrt=T){
> if (abs(sum(pi)-1)>0.01) warning("sum of pi must be 1")
> n    <- length(yi); yfrq <- rep(1,n)
> if (ysrt){
>   iord <- order(yi); yi <- yi[iord]; pi <- pi[iord]}
> iu   <- 1; yu <- yi; pu <- pi
> dup  <- duplicated(yu)
> n    <- length(yi)
> for(i in 2:n){
>  if(dup[i]){
>     pu[iu] <- pu[iu] + pi[i]
>     yfrq[iu] <- yfrq[iu]+1}
>  else{
>    iu     <- iu+1
>    yu[iu] <- yi[i]
>    pu[iu] <- pi[i]}}
> yu   <- yu[1:iu]; pu <- pu[1:iu]; yfrq <- yfrq[1:iu]
> list(yu=yu, pu=pu, yfrq=yfrq)}
```

References

Marazzi A. (1993). Algorithms, Routines, and S functions for Robust Statistics. Chapman and Hall, New York.

Marazzi A., Randriamiharisoa A. (1997). S-Plus functions for robust estimators of the parameters of the gaussian and lognormal distributions.

Marazzi A., Ruffieux C. (1998). Rules for removing outliers and tests based on parametric models for estimating and comparing average length of stay. Patient Classification Systems/Europe (PCS/E), International Working Conference Proceedings Manual, 1-3 October 1998, Manchester.

Marazzi A., Ruffieux C. (1999). The truncated mean estimate of the mean of an asymmetric distribution. Computational Statistics & Data Analysis, 32(1), 79-100.

Marazzi A. (2001). Bootstrap procedures for robust means of asymmetric distributions. Computational Statistics & Data Analysis. In press.